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Exhibit C – Part I

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Madoff - Proposal

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This work benefitted greatly from discussions with Nat, Jim, Henry, and Paul. The desk's conversations with OEX option brokers was also useful. Thanks to Henry for suggesting the research.

1 Executive Summary

The executive summary is that there is a fair understanding of the components that go into the Madoff strategy, even though some conclusions are still lacking in rigor. On a positive note, we propose a pilot program combining research and production, where the end results are a trading strategy that could take at first glance up to \$.5B portfolio of stocks and generate returns of at least 7-8%/year.

2 Introduction

Starting from paper printouts of a managed account an attempt was made to understand the trading strategy behind the account and to see if we can benefit from some analogous strategy. Six years of accounted trades were transcribed to the futures file format and analyzed. These files include option, stock, Treasury Bill, and Money market transactions. In order to explore further, a simulator was written that allowed for variations in the basic trading style and also allowed experiments to be carried out over a longer time span than the transcribed data.

This report is divided into six sections. In the third section there is an qualitative summary of the salient points of the researched strategy and what we have learned about it. In the fourth section there is a more detailed statistical description of how we arrived at these conclusions and in section five some of the experiments carried out using our Madoff simulator are discussed. In Section six we address the issue of where we go from here.

3 The Madoff Strategy

The trading data we transcribed goes from Oct. 95 until the end of 2001. Although data is available until the present day it was decided not to include it in our analysis and to reserve this period for possible out of sample studies. Investigating his accounting sheets reveals Madoff does not borrow or lend, is fully invested for the period he is in the market and does not lever his position. All profit and loss stated can be accounted for directly from the given transaction sheets. Madoff returns around 18% per year including Treasury interest.

The Qualitative Summary of the Strategy:

- 1) The strategy is quite straightforward: a basket of stocks is bought and an OEX option collar is applied. This collar consists of a long put and a short call where the strike price of the call is slightly higher than the put. Madoff holds his position for an average of 28 days and for the 6 years for which we have accounting sheets there are 34 cycles. The stock and option position is fully unwound and the money put into Treasury's until the next cycle. Madoff is invested in the stock and option market about 60% of the available time.
- 2) Analysis suggests Madoff's stock transactions occur within one day. For the dates we analyzed, there are only 4 individual stock transactions that fall outside of the low/high range on a given trading day. As far as we can tell from the price he paid for the options they are mostly bought or sold on the stock transaction day, but we note the call is quite often sold on the day after the put is bought. Very recently, when the amount under management has increased he may put on his stock position over a couple of days but if this is so he will put on a different collar each day in proportion to that days trading. We also note that secured returns for a second managed account suggest all trades took place on the same day and in proportion to the amount

in the managed account. Moreover comparison of these two managed accounts show the same stock/option portfolio is used.

- 3) The put option is almost always bought at a strike price that is slightly lower or equal to the 4pm value of the OEX index. The sold call typically has a strike price 10-15 higher. Often, the stock/option position is unwound just before or at the option expiry date (3rd Friday of month) but sometimes there is a roll of the option contracts to the next month. Integrating the cost for the put/call options over the life of available data shows this collar is of very little cost i.e. the profit from selling the call approximately equals the cost of buying the put. We have investigated this feature in more detail and concluded the put/call prices that Madoff quotes are consistent with the days market range. However, we are impressed with his timing for selling the call (including delaying trading by one day) and have asked the desk to keep a log of OEX put/call prices near 4pm.
- 4) The stock position is typically the first 30-35 stocks of the OEX 100 index. The portfolio is divided into dollar amounts per stock that correspond to the ratio each stock contributes to the OEX capital weighted index. These stocks correspond to 80% of the total OEX index value and are correlated with the index at the 98% level.
- 5) Madoff achieves spectacular fills on his stock portfolio. Just under a half of his reported profit may be attributed to the difference between the close price and the buy/sell price. This profit corresponds to about 43c/share. Madoff achieves better fills on the buy side than the sell side.
- 6) There is a mystery as to how Madoff hides his trades and how much he has under management. For instance, if we take the prevailing wisdom that he has approximately \$6 billion under management then using early 2002 OEX index values we would need approximately 133000 put or call contracts for the collar. Daily volume figures for the exchange traded OEX options do not support anything like this trade size and unlike SPX options the OTC market in OEX options is not developed. This raises several interesting questions: either the fund is much smaller than we believe (internet hedgefund summary site gives Madoff

- at \$400 million which supports the numbers we see), or the reported trade figures are some kind of average over several days of trading, or our inquires into the size of the OTC market are wrong.
- 7) The option collar reduces his p&l relative to a stock only position but increases the sharpe ratio of the portfolio. The options only seem to be used for risk control.
- 8) There is mild evidence for market timing effects; for instance running the simulator (described in section four) and comparing Madoff's daily returns with mirror simulations, that put him in the market for those dates that the trading data say he is out, give a t-value of 2.5sigma (stock fill price average of daily high/low).
- 9) Madoff is not completely hedged with the portfolio that is put on. The liquidity of the top 30-35 stocks is used to get into position and the bottom 65 are not hedged during the remaining period. Of course this will contribute to his profit/loss.

In summary Madoff makes a third of his money buying a basket of stocks that are very strongly correlated with the OEX index at a great fill price (relative to the close price) and then putting on a collar with a small spread at essentially no cost. Additional profit comes from the stocks gain between the bought put and sold call strike price or possible mispricing between the stock basket and options. Finally, good sell prices (relative to the close) and TBills make up the rest of the returns.

Normally it would be difficult to buy a stock basket and buy an at the money put using the money gained by selling a call just out of the money because the put would be far too expensive. On analyzing how we might achieve this cost free collar in practice, the cash index would have to be approximately half way between the put and call strike prices. We say approximately half way because of the option skew and the market spread. Madoff is most likely gambling his initial long stock portfolio will go up and then he puts on the collar. This gamble on the stocks rising may be the result of market predictions or he has a customer who is willing to consistently sell him a basket of stocks below the current market value.

To understand the former, the challenge is to see if we can find a rising market predictor based around Madoff's trading dates.

4 Detailed Statistical Description

• 1) Graphical Representation of Madoff Profits

The following graphs represent the accumulative daily profits of the Simons foundation investment in the Madoff fund. These daily profits were generated by combining on trading days the prices from Madoff's accounting sheets with daily closing values for stocks, options, and T-Bills from our data base. The Simons foundation added various capital for the time interval of interest here.

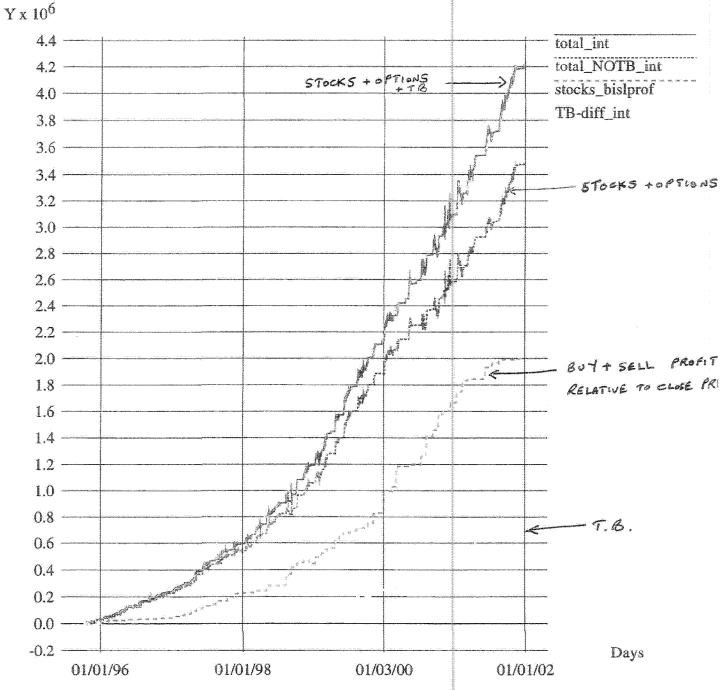
Graph 1. Comparison of total accumulative Madoff profits (stocks + options + T-Bills) with profits of stocks+options only and T-Bills only. The combined buy+sell profits for the stocks relative to the close price for those trading days are also shown.

Graph 2. More detailed graph of the components leading to the accumulative Madoff profit stream. Unlike Graph 1, we show here stock only profits (stocks_int), call profits (optionsC_int), put profits (optionsP_int), and profits from buying (stocks_biprof_int) and selling (stocks_slprof_int) the stocks on the trading days relative to the closing price.

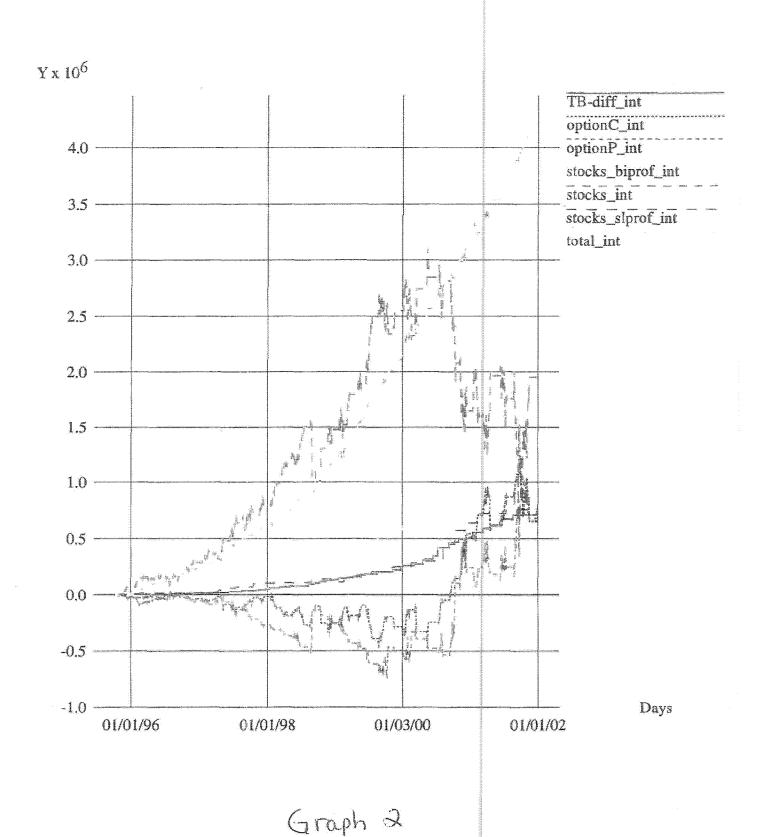
Graph 3. This graph shows amongst others the residual profit total_NOTB_int after subtracting the TBill profit and fill profit(relative to close) from total Madoff profit.

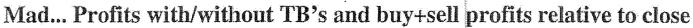
Graph 4. Graph for the accumulated cost (via trading prices) of the put/call option collar.

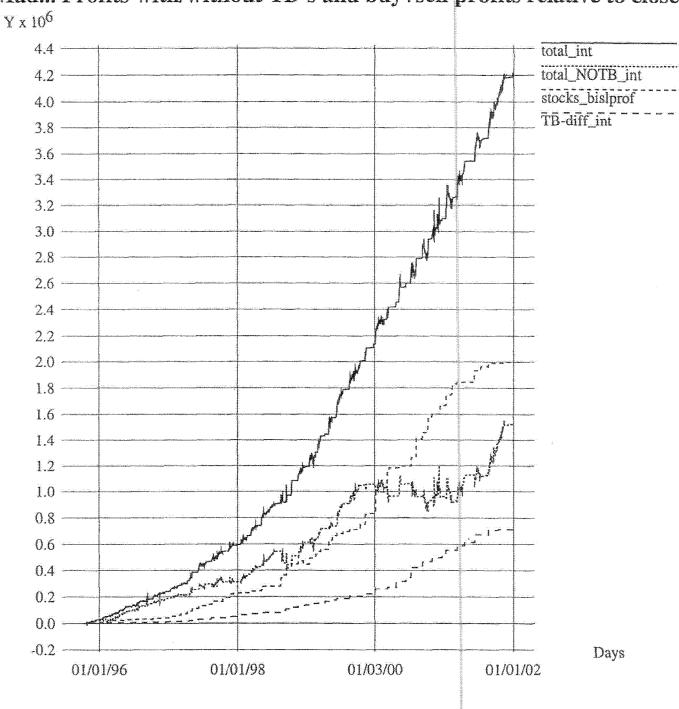
Mad... Profits with/without TB's and buy+sell profits relative to close



Graph 1

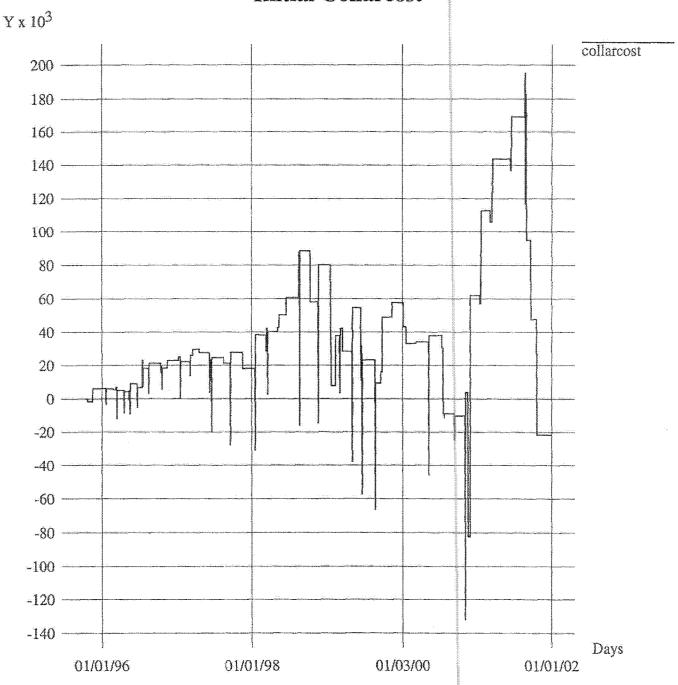






Graph 3

Initial Collarcost



Graph 4

• 2) More Detailed Statistical Analysis of the Madoff Strategy

The following few pages summarize statistical analysis of the actual Madoff daily returns of the Simons foundation. The daily returns take into account the additional monies added to the account.

There are comments throughout the output to clearly indicate the nature of the two time series being compared and comments for the interpretation of the output.

2a) Compare stocks daily return for Madoff fill prices and the close prices.

X-file is stocks (Madoff Fills)
Y-file is stocks_cl (Close Prices)

sday is 10/18/1995 eday is 12/31/2001

Number of valid points: 1618 Number of invalid points: 1

X-mean: 0.0556413 X-sd: 0.835351 X-m/s: 0.0666083

X-max: 4.68763 on 10/15/1998 X-min: -6.69193 on 08/31/1998

Y-mean: 0.0291249 Y-sd: 0.829023 Y-m/s: 0.0351316

Y-max: 4.68763 on 10/15/1998 Y-min: -7.23953 on 08/31/1998

Corr: 0.978741 Nip: 0.978308

2b) Comparing difference of stock returns for Madoff fills and close prices

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X-file is ddiff sday is 10/18/1995 eday is 12/31/2001 Number of valid points: 1618 Number of invalid points: 1

X-mean: 0.0265161 X-sd: 0.171713 X-m/s: 0.154421

X-max: 3.18231 on 01/07/2000
X-min: -0.600764 on 12/17/1998

Corr: 0.0133806 Nip: 0.138795

** NOTE A 6.2 SIGMA SIGNIFICANCE FOR MADOFF FILLS OVER CLOSE PRICES !! **

2c) Compare daily returns of option and stocks (Madoff Fills)

partial is options + stocks return is options plus stocks plus TBill interest

X-file is return
Y-file is partial
sday is 10/18/1995 eday is 12/31/2001
Number of valid points: 1618
Number of invalid points: 1

X-mean: 0.0646938 X-sd: 0.352584 X-m/s: 0.183485

X-max: 3.1515 on 09/16/1998 X-min: -3.23754 on 09/17/1998

Y-mean: 0.0520354 Y-sd: 0.344982 Y-m/s: 0.150835

Y-max: 3.1515 on 09/16/1998

Y-min: -3.23754 on 09/17/1998

Corr: 0.983188 Nip: 0.983144

X-file is stocks Y-file is option

sday is 10/18/1995 eday is 12/31/2001

Number of valid points: 1618 Number of invalid points: 1

X-mean: 0.0556413 X-sd: 0.835351 X-m/s: 0.0666083

X-max: 4.68763 on 10/15/1998 X-min: -6.69193 on 08/31/1998

Y-mean: -0.00360587 Y-sd: 0.735466 Y-m/s: -0.00490285

Y-max: 6.5383 on 08/31/1998 Y-min: -3.99245 on 09/13/1996

Corr: -0.911263 Nip: -0.909563

** NOTE The options do not contribute to overall profits. **

2d) Difference between daily stocks return (Madoff Fills) and return of S500

X-file is dddiff

sday is 10/18/1995 eday is 12/31/2001

Number of valid points: 918 Number of invalid points: 701 X-mean: 0.0715412
X-sd: 0.477625
X-m/s: 0.149785

X-max: 3.63429 on 03/12/2001 X-min: -2.39355 on 12/11/2000

Corr: 0.024856 Nip: 0.137184

** NOTE A 4.5 sigma significance for stocks return over S500 with Madoff fills **

2e) Here are serial correlations of various components of Madoff's profits.

1 is 1 ago diff is return - partial

X-file is return Y-file is return1 sday is 10/19/1995 eday is 12/31/2001 Number of valid points: 1617 Number of invalid points: 1

X-mean: 0.0646286 X-sd: 0.352683 X-m/s: 0.183249

X-max: 3.1515 on 09/16/1998 X-min: -3.23754 on 09/17/1998

Y-mean: 0.0647338 Y-sd: 0.352689 Y-m/s: 0.183544

Y-max: 3.1515 on 09/17/1998 Y-min: -3.23754 on 09/18/1998

Corr: -0.202572

Nip: -0.163441

X-file is stocks Y-file is stocks1

sday is 10/19/1995 eday is 12/31/2001

Number of valid points: 1617 Number of invalid points: 1

X-mean: 0.0556757 X-sd: 0.835608 X-m/s: 0.066629

X-max: 4.68763 on 10/15/1998 X-min: -6.69193 on 08/31/1998

Y-mean: 0.0556757 Y-sd: 0.835608 Y-m/s: 0.066629

Y-max: 4.68763 on 10/16/1998 Y-min: -6.69193 on 09/01/1998

Corr: 0.0322388 Nip: 0.0365161

X-file is option Y-file is option1

sday is 10/19/1995 eday is 12/31/2001

Number of valid points: 1617 Number of invalid points: 1

X-mean: -0.0036081
X-sd: 0.735693
X-m/s: -0.00490436

X-max: 6.5383 on 08/31/1998 X-min: -3.99245 on 09/13/1996

Y-mean: -0.0036081 Y-sd: 0.735693

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Y-m/s: -0.00490436

Y-max: 6.5383 on 09/01/1998 Y-min: -3.99245 on 09/16/1996

Corr: 0.022097 Nip: 0.0221206

X-file is partial Y-file is partial1

sday is 10/19/1995 eday is 12/31/2001

Number of valid points: 1617 Number of invalid points: 1

X-mean: 0.0520676 X-sd: 0.345086 X-m/s: 0.150883

X-max: 3.1515 on 09/16/1998 X-min: -3.23754 on 09/17/1998

Y-mean: 0.0520676 Y-sd: 0.345086 Y-m/s: 0.150883

Y-max: 3.1515 on 09/17/1998 Y-min: -3.23754 on 09/18/1998

Corr: -0.207547 Nip: -0.180668

X-file is diff Y-file is diff1

sday is 10/19/1995 eday is 12/31/2001

Number of valid points: 1617 Number of invalid points: 1

X-mean: 0.0125611 X-sd: 0.0643035 X-m/s: 0.19534

X-max: 0.931492 on 07/13/2000 X-min: -6.38378e-16 on 08/31/1998

Y-mean: 0.0126662 Y-sd: 0.0644217 Y-m/s: 0.196614

Y-max: 0.931492 on 07/14/2000 Y-min: -6.38378e-16 on 09/01/1998

Corr: -0.000771063 Nip: 0.0362435

X-file is option Y-file is stocks1

sday is 10/19/1995 eday is 12/31/2001

Number of valid points: 1617 Number of invalid points: 1

X-mean: -0.0036081
X-sd: 0.735693
X-m/s: -0.00490436

X-max: 6.5383 on 08/31/1998 X-min: -3.99245 on 09/13/1996

Y-mean: 0.0556757 Y-sd: 0.835608 Y-m/s: 0.066629

Y-max: 4.68763 on 10/16/1998 Y-min: -6.69193 on 09/01/1998

Corr: -0.0625131 Nip: -0.0627001

X-file is option1 Y-file is stocks

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sday is 10/19/1995 eday is 12/31/2001

Number of valid points: 1617 Number of invalid points: 1

X-mean: -0.0036081
X-sd: 0.735693
X-m/s: -0.00490436

X-max: 6.5383 on 09/01/1998 X-min: -3.99245 on 09/16/1996

Y-mean: 0.0556757 Y-sd: 0.835608 Y-m/s: 0.066629

Y-max: 4.68763 on 10/15/1998 Y-min: -6.69193 on 08/31/1998

Corr: -0.0337632 Nip: -0.0340141

2f) The files in and out correspond to change in SAP future when stock p&l is non-zero or zero, respectively.

X-file is out

sday is 10/18/1995 eday is 12/31/2001

Number of valid points: 700 Number of invalid points: 919

X-mean: 0.0227377
X-sd: 1.26685
X-m/s: 0.0179483

X-max: 5.50904 on 10/28/1997 X-min: -7.75753 on 10/27/1997

Corr: -0.0313934 Nip: 0.00298825 X-file is in Y-file is stocks sday is 10/18/1995 eday is 12/31/2001

Number of valid points: 918 Number of invalid points: 701

X-mean: 0.0265281 X-sd: 1.19926 X-m/s: 0.0221203

X-max: 5.13428 on 10/15/1998 X-min: -7.91544 on 08/31/1998

Y-mean: 0.0980693 Y-sd: 1.10714 Y-m/s: 0.0885792

Y-max: 4.68763 on 10/15/1998 Y-min: -6.69193 on 08/31/1998

Corr: 0.917289 Nip: 0.91544

X-file is in Y-file is option

sday is 10/18/1995 eday is 12/31/2001

Number of valid points: 918 Number of invalid points: 701

X-mean: 0.0265281 X-sd: 1.19926 X-m/s: 0.0221203

X-max: 5.13428 on 10/15/1998 X-min: -7.91544 on 08/31/1998

Y-mean: -0.00651655 Y-sd: 0.976353 Y-m/s: -0.00667438

Y-max: 6.5383 on 08/31/1998 Y-min: -3.99245 on 09/13/1996 Corr: -0.914512 Nip: -0.914415

X-file is in Y-file is partial

sday is 10/18/1995 eday is 12/31/2001

Number of valid points: 918 Number of invalid points: 701

X-mean: 0.0265281
X-sd: 1.19926
X-m/s: 0.0221203

X-max: 5.13428 on 10/15/1998 X-min: -7.91544 on 08/31/1998

Y-mean: 0.0915527 Y-sd: 0.453949 Y-m/s: 0.201681

Y-max: 3.1515 on 09/16/1998 Y-min: -3.23754 on 09/17/1998

Corr: 0.270247 Nip: 0.26922

X-file is out Y-file is partial

sday is 10/18/1995 eday is 12/31/2001

Number of valid points: 700 Number of invalid points: 919

X-mean: 0.0227377
X-sd: 1.26685
X-m/s: 0.0179483

X-max: 5.50904 on 10/28/1997

X-min: -7.75753 on 10/27/1997

Y-mean: 0.000211272 Y-sd: 0.0105033 Y-m/s: 0.0201148

Y-max: 0.271344 on 09/03/2001 Y-min: -0.0392438 on 05/09/1997

Corr: -0.0105021 Nip: -0.0101373

X-file is in Y-file is return

sday is 10/18/1995 eday is 12/31/2001

Number of valid points: 918 Number of invalid points: 701

X-mean: 0.0265281
X-sd: 1.19926
X-m/s: 0.0221203

X-max: 5.13428 on 10/15/1998 X-min: -7.91544 on 08/31/1998

Y-mean: 0.0994954 Y-sd: 0.45979 Y-m/s: 0.216393

Y-max: 3.1515 on 09/16/1998 Y-min: -3.23754 on 09/17/1998

Corr: 0.265874 Nip: 0.264473

X-file is out Y-file is return sday is 10/18/1995 eday is 12/31/2001

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Number of valid points: 700 Number of invalid points: 919

X-mean: 0.0227377 X-sd: 1.26685 X-m/s: 0.0179483

X-max: 5.50904 on 10/28/1997 X-min: -7.75753 on 10/27/1997

Y-mean: 0.0190539 Y-sd: 0.0801834 Y-m/s: 0.237629

Y-max: 0.931492 on 07/13/2000 Y-min: -0.0367385 on 05/09/1997

Corr: 0.0193597 Nip: 0.022981

X-file is inOption Y-file is stocks

sday is 10/18/1995 eday is 12/31/2001

Number of valid points: 918 Number of invalid points: 701

X-mean: -0.00651655
X-sd: 0.976353
X-m/s: -0.00667438

X-max: 6.5383 on 08/31/1998 X-min: -3.99245 on 09/13/1996

Y-mean: 0.0980693 Y-sd: 1.10714 Y-m/s: 0.0885792

Y-max: 4.68763 on 10/15/1998 Y-min: -6.69193 on 08/31/1998 Corr: -0.912594 Nip: -0.909603

5 Simulator

To better understand the Madoff strategy, we wrote a simulator that allows for variations on the basic theme.

Description of the Madoff Simulator

At the present time the Madoff simulator does not use a prediction to put on or take off a position. However, for a given risk capital, the simulator will put on a stock position of the first n stocks (n can be specified) of the OEX 100 in proportion to yesterdays capital weightings at the close. In addition the option collar is put on in direct proportion to the stock position using the 4pm OEX mid-quote as a reference. The sequence of entry/exit days can be varied as can the width of the option spread, the number of stocks that make up the portfolio and the price at which the stocks are bought or sold i.e. open/high/low/close etc. The option collar itself can also be left off if desired. In this way we have a number of variables to investigate the Madoff strategy and can extend the simulation beyond the relatively short time we have real trade data available. Note that our simulator does not invest in T-bills when we are out of the market and we always start with \$1 million dollars invested.

• Some results from the Madoff Simulator

Graph 5 Comparison of profit curves for Madoff dates and continuously holding the stock portfolio (portpl15 and portpl15_cont_av respectively). For portpl15_cont_av the options are rolled at expiry day and for both curves the put/call option spread is 15. The stocks are filled at the average of the high/low for the day.

Graph 6 Comparison of profit curves for Madoff dates and continuously holding the stock portfolio (portpl15_cl and portpl15_cont_cl respectively). For portpl15_cont_cl the options are rolled at expiry day and